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A SECRETARY PROBLEM WITH A RANDOM NUMBER OF CHOICES.(U)  
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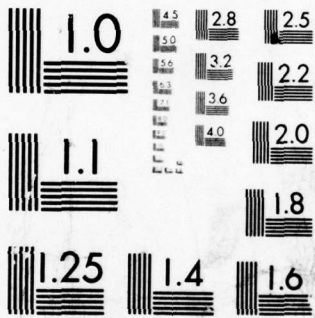
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⑩ Kenneth S. Glasser

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JOB

A SECRETARY PROBLEM  
WITH A RANDOM NUMBER OF CHOICES

by

Kenneth S. Glasser

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ABSTRACT

In this paper, the best-choice Secretary Problem is modified to allow the player to make more than one choice. The probability of selecting the best object is computed. The optimal starting time is characterized, and expressions are derived for the expected number of objects chosen and the expected number of objects sampled by this procedure. Asymptotic results are also derived.

KEY WORDS: Secretary Problem, Optimal Stopping Rules, Relative Ranks.

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## 1. INTRODUCTION

In this article, we will consider a procedure that is closely related to the optimal procedure for the best choice Secretary Problem. The problem is formulated as follows:  $N$  objects or individuals can be ranked from best (rank=1) to worst (rank= $N$ ). As each object is shown to the experimenter (or player), he is able to rank it only in relation to those objects that he has already seen. The player may select the current object, at which point the procedure ends, or he may elect to reject the current object and sample the next. Once an object has been rejected, it may not be chosen later. If no choice has been made, the  $N^{\text{th}}$  object must be chosen.

In the best choice Secretary Problem, the player tries to maximize the probability of selecting the best object. The optimal procedure, first derived by Lindley (1961), has the following form: Beginning with the  $r^{\text{th}}$  object, choose the first object that is better than all others seen so far. Following Gilbert and Mosteller (1966), we shall call such an object a candidate.  $r$  is called the starting time.

The optimal starting time,  $r^*$ , is given by:

$$r^* = \min \left\{ r \mid \sum_{k=r}^{N-1} (1/k) < 1 \right\}. \quad (1.1)$$

Tables of  $r^*$  and the probability of success for this procedure are given in Gilbert and Mosteller (1966), for  $N=1(1)50(10)100,1000$ .



As  $N$  becomes large, the probability of choosing the best object using the optimal procedure decreases to  $1/e$ , or less than 0.37. It is reasonable to ask if this probability of success can be increased if some of the restrictions placed on the problem are relaxed. The following procedure is proposed to improve the success rate. Note that by using this procedure, it is quite possible for the player to choose more than one object.

Procedure A: Do not select any of the first  $s-1$  objects. Choose the next candidate encountered. Sample the next object; if it is a candidate, select it also. Continue sampling until either a noncandidate or the  $N^{\text{th}}$  object appears. If no objects have been chosen, the  $N^{\text{th}}$  object must be selected.

When the player succeeds in choosing the best object, it will be called a correct selection (CS). The probability of success using Procedure A will be given by  $P_A(\text{CS}; s, N)$ , or by  $P_A(\text{CS})$  when it is clear that  $s$  and  $N$  are given fixed values. The player chooses  $s$  to maximize the probability of having the best object among those selected.

Procedure A is equivalent to the finite memory Secretary Problem (FM) of Smith and Deely (1975) when  $m=2$ , in that the probability of success is the same in both cases. FM gives the player only one choice, however. Also, the definition of starting times are somewhat different; the optimal starting time of FM is always one greater than that of Procedure A. Finally,

FM can be generalized by allowing  $m$  to vary, as Smith and Deely have done. Procedure A will be generalized in Section 5 by allowing the player to encounter several noncandidates before he is forced to stop the procedure.



## 2. COMPUTING THE PROBABILITY OF SUCCESS

We begin by defining several symbols and events. First, the arrival time of the best object will be denoted by  $a^*$ . For example,  $a^*=5$  means that the best item was sampled fifth.  $a_k$  and  $b_k$  will denote the arrival times of the relatively best and second best objects respectively, in the first  $k$  sampled.

The player succeeds when candidates are chosen at times  $k, k+1, \dots, j-1$  and the best object is chosen at time  $j$ . This event will be denoted by  $W(k, j)$ , where  $k \geq s$ .  $W(k, j)$  can occur only if the second best of the first  $k$  items seen has been observed before  $s$ . In terms of the above notation, we can write  $W(k, j)$  as:

$$W(k, j) = [b_k < s, a_k = k, a_{k+1} = k+1, \dots, a_{j-1} = j-1, a^* = j] .$$

The probability that  $W(k, j)$  occurs may be computed by first noting:

$$P(W(k, j)) = P(b_k < s, a_k = k, \dots, a_{j-1} = j-1 | a^* = j) P(a^* = j). \quad (2.1)$$

The probability that the best object arrives at any particular time  $j$  is given by  $1/N$ , for  $j=1, \dots, N$ . Now, assume that  $s > 1$ . Given that the best object arrives at time  $j$ , the only restriction on the first  $k-1$  objects is that the best (the second best out of  $k$ ) must arrive before  $s$ . So there are  $k-2$  objects that can be permuted (in  $(k-2)!$  ways) in any order, without affecting the outcome of  $W(k, j)$ . Also, the best object

of the first  $k-1$  may arrive any time before  $s$ , or in any of  $s-1$  times. There are  $(j-1)!$  possible permutations of the first  $j-1$  objects. Combining this in (2.1) yields, for  $k=s, \dots, N$  and  $j=k, \dots, N$ :

$$P(W(k,j)) = (s-1) \cdot f(k-2, j-1) / N, \quad (2.2)$$

$$\text{where} \quad f(k,j) = k! / j! \quad (2.3)$$

When  $s=1$ , the first object is automatically selected, since it is by definition a candidate. The player will win only if every object sampled is a candidate until the best object appears. Thus:

$$P(W(1,k)) = f(0, k-1) / N. \quad (2.4)$$

Summing (2.2) and (2.4) over all possible values of  $j$  and  $k$  yields:

$$\begin{aligned} P_A(CS; s, N) &= \frac{1}{N} \sum_{k=1}^N f(0, k-1), \quad \text{if } s=1 \\ &= \frac{s-1}{N} \sum_{k=s}^N \sum_{j=k}^N f(k-2, j-1), \quad \text{if } s>1. \end{aligned} \quad (2.5)$$

We next calculate the expected number of objects chosen using Procedure A. First, let  $s>1$ .

The player selects objects arriving at  $k, k+1, \dots, j$ , if and only if all of these objects are candidates and the  $(j+1)^{\text{st}}$  object is not a candidate. If this occurs,  $j-k+1$  objects are chosen. For any  $i$ , the probability that the  $i^{\text{th}}$  object is a

candidate is  $1/i$ ,  $i=1, \dots, N$ . The probability that the  $i^{\text{th}}$  object is not a candidate is then  $(i-1)/i$ . Since the objects appear in random order, the relative rank of the  $i^{\text{th}}$  object is independent of the ranks of the first  $i-1$  objects. Thus,

$$P(a_k=k, \dots, a_j=j=a_{j+1}) = j \cdot f(k-1, j+1). \quad (2.6)$$

The arrival time of the best object of the first  $k-1$  is independent of the relative ranks of the objects arriving at times  $k, k+1, \dots, j+1$ , so that:

$$P(b_k < s | a_k=k, \dots, a_j=j=a_{j+1}) = \frac{s-1}{k-1}. \quad (2.7)$$

Finally, the probability of selecting  $j-k+1$  objects is given by multiplying (2.6) and (2.7):

$$P(b_k < s, a_k=k, \dots, a_j=j=a_{j+1}) = (s-1) \cdot f(k-2, j-1) / (j+1), \quad (2.8)$$

for  $j = k, k+1, \dots, N-1$ , and  $k = s, \dots, N-1$ .

The probability of selecting the best object at time  $N$ , when the first choice was made of a candidate arriving at time  $k$ , is given by:

$$P(b_k < s, a_k=k, \dots, a_{N-1}=N-1, a^*=N) = (s-1) \cdot f(k-2, N). \quad (2.9)$$

Equation (2.9) describes an event where  $N-k+1$  objects are chosen.

If no candidates are encountered after time  $s-1$ , the player must choose the  $N^{\text{th}}$  object. He will make one choice of



a noncandidate at time  $N$  with probability:

$$P(a^* < s) = (s-1)/N. \quad (2.10)$$

Denote the number of objects chosen by the random variable  $X$ . The expected value of  $X$  is given by multiplying (2.8), (2.9), and (2.10) by the number of objects chosen and then summing over their respective values of  $j$  and  $k$ :

$$\begin{aligned} E_A(X; s, N) = & (s-1) \sum_{k=s}^{N-1} \sum_{j=k}^{N-1} \frac{j-k+1}{j+1} f(k-2, j-1) \\ & + (s-1) \sum_{k=s}^N (N-k+1) f(k-2, N) + \frac{s-1}{N}, \end{aligned} \quad (2.11)$$

for  $s > 1$ .

For  $s=1$ , the equations are simpler in form. We can compute:

$$E_A(X; 1, N) = \sum_{k=1}^N k^2 f(0, k+1) + f(0, N-1). \quad (2.12)$$

The  $m^{\text{th}}$  moment can be calculated in the same manner as (2.11) and (2.12).

We can also calculate the expected number of objects sampled before the procedure ends. First, for  $s > 1$ , the probability that  $j+1$  objects are sampled when the first choice is made at  $k \geq s$  is given by (2.8). The probability that  $N$  objects are sampled when the first choice is made at  $k$  is given by (2.9). The probability that  $N$  objects are sampled, and the

only choice is made of a noncandidate at  $N$  is given by (2.10). Denote the number of objects sampled by the random variable  $Y$ . Then, the expected value of  $Y$  is given by:

$$\begin{aligned}
 E_A(Y; s, N) &= \sum_{k=s}^{N-1} \sum_{j=k}^{N-1} (j+1) \frac{s-1}{j+1} f(k-2, j-1) \\
 &\quad + \sum_{k=s}^N (s-1) \cdot N \cdot f(k-2, N) + N \cdot \frac{s-1}{N} \\
 &= N P_A(CS; s, N) + (s-1). \tag{2.13}
 \end{aligned}$$

It can be shown that (2.13) also holds for  $s=1$ .



### 3. FINDING THE OPTIMUM S

Equation (2.5) gives the probability of success using Procedure A. The following lemma shows the existence of a starting point  $s^*$  that maximizes  $P_A(CS)$  for a given  $N$ .

Lemma 3.1: There exists an  $s$ ,  $1 \leq s \leq N$ , that maximizes  $P_A(CS)$  for a fixed  $N$ . For any  $s' < s$ ,  $P_A(CS; s', N) \leq P_A(CS; s, N)$ . For any  $s' > s$ ,  $P_A(CS; s, N) \geq P_A(CS; s', N)$ .

Proof: For any  $s$ ,

$$\begin{aligned} F(s; N) &= N \left[ P_A(CS; s, N) - P_A(CS; s+1, N) \right] \\ &= 1 - \sum_{k=s+2}^N \sum_{j=k}^N f(k-2, j-1). \end{aligned} \tag{3.1}$$

$F(s; N)$  is a non-decreasing function of  $s$ , since:

$$\begin{aligned} F(s+1; N) - F(s; N) &= \sum_{j=s+2}^N f(s, j-1) \\ &\geq 0. \end{aligned}$$

Since  $F(s; N)$  is non-decreasing in  $s$ , if  $F(s; N) \geq 0$ , then  $F(s+1; N) \geq 0$ . This completes the proof.

The optimum  $s$  for a given  $N$  is found by using (3.1), since  $P_A(CS;s,N) > P_A(CS;s+1,N)$  if  $F(s;N) > 0$ . Thus, the  $s$  that maximizes  $P_A(CS;s,N)$  is given by:

$$s^* = \min \left[ s \mid 1 > \sum_{k=s+2}^N \sum_{j=k}^N f(k-2, j-1) \right]. \quad (3.2)$$

In practice, (3.2) does not substantially reduce the amount of work necessary to find  $s^*$ . The double sum in (3.2) can be computed using backwards iteration, since:

$$\sum_{k=s}^N \sum_{j=k}^N f(k-2, j-1) = \sum_{j=s}^N f(s-2, j-1) + \sum_{k=s+1}^N \sum_{j=k}^N f(k-2, j-1). \quad (3.3)$$

When  $F(s;N) = 0$ , from (3.1) it can be seen that  $P_A(CS;s,N) = P_A(CS;s+1,N)$ . In this case,  $s^*$  could be taken to be either  $s$  or  $s+1$ . We assume that the player will wish to take the  $s^*$  that gives the lower value of  $E_A(X;s,N)$ .

Table 1 contains values of  $s^*$ ,  $P_A(CS;s^*,N)$ ,  $E_A(X;s^*,N)$ , and  $E_A(Y;s^*,N)$  for  $N = 1(1) 50(10) 100, 1000$ . For small  $N$ , the probability of success is quite a bit better than that of the optimal procedure of the best-choice Secretary Problem. As  $N$  becomes large, however, there is not much difference. This is because as  $s^*$  increases, the probability of making more than one choice decreases. The next section will cover asymptotic results in detail.

Also, as mentioned in section 2.1, some of the results can be compared with certain results obtained by Smith and Deely (1975). In their table (p.361), for  $m=2$  and  $N=10,50,\text{and }100$ , the probabilities of success agree exactly. Note that their optimal starting time  $r^*$  is always one greater than the optimal starting time  $s^*$  derived in this section. This is due to a difference in the definitions of  $r^*$  and  $s^*$ .



#### 4. ASYMPTOTIC RESULTS

In this section, we will study  $P_A(CS)$ ,  $E_A(X)$ , and  $E_A(Y)$  as  $N$  increases to infinity. First, let  $s^*(N)$  be the optimal starting time when picking the best of  $N$  objects using Procedure A. When there is no ambiguity,  $s^*(N)$  will be written as  $s^*$ .

Lemma 4.1       $s^*(N+1) \geq s^*(N)$  .

Proof:          For any  $r < s^*(N)$ :

$$\sum_{k=r+2}^{N+1} \sum_{j=k}^{N+1} f(k-2, j-1) > \sum_{k=r+2}^N \sum_{j=k}^N f(k-2, j-1) > 1 .$$

The last inequality follows from the definition of  $s^*(N)$  in (3.2) and since  $r < s^*(N)$  .

Lemma 4.2       $\lim_{N \rightarrow \infty} s^*(N) = \infty$  .

Proof:          We will prove this lemma by showing that there exists an  $N_2 > N_1$  large enough so that  $s^*(N_2) > s^*(N_1)$ . Note first, that for any  $s$  :

$$\sum_{k=s+2}^N \sum_{j=k}^N f(k-2, j-1) > \sum_{k=s+2}^N 1/(k-1) . \tag{4.1}$$

As  $N$  increases to infinity, the right hand side of (4.1) diverges. For  $N=N_1$  and  $s=s^*(N_1)$ , the left hand side of (4.1) must be less than one, by (3.2), but there are an infinite number of  $N_2$ 's large enough to make the entire inequality greater than one. Define:

$$N_2 = \min \left[ M \mid \sum_{k=s^*(N_1)+2}^M 1/(k-1) > 1 \right].$$

Then  $s^*(N_2) > s^*(N_1)$ .

Corollary 4.1  $\lim_{N \rightarrow \infty} E_A(Y) = \infty$ .

Proof: This result follows directly from Lemma 4.2, since the player must wait at least until  $s^*(N)$  to make his first choice.

Theorem 4.1:  $\lim_{N \rightarrow \infty} (s^*/N) = \lim_{N \rightarrow \infty} P_A(CS; s^*, N) = 1/e$ . (4.2)

Proof: It is well known (Lindley (1961), etc.) that (4.2) holds for the optimal procedure for the best-choice Secretary Problem. We will prove this theorem by showing that the optimal procedure and Procedure A are asymptotically equivalent. Thus, (4.2) holds for Procedure A also.

For  $s > 1$ , the probability that one object is chosen at  $k$  is given by:

$$P \left[ b_k < s, a_k = k = a_{k+1} \right] = \frac{s-1}{(k-1)(k+1)}, \quad k=s, \dots, N-1.$$



One candidate is chosen at time  $N$  with probability:

$$P(b_N < s, a^* = N) = \frac{s-1}{(N-1)N} .$$

Finally, one object is chosen if the player is forced to choose a non-candidate at time  $N$ . The probability that this occurs is given by equation (2.10).

Thus,

$$\begin{aligned} P(X=1; s, N) &= (s-1) \left[ \sum_{k=s}^{N-1} \frac{1}{(k-1)(k+1)} + \frac{1}{(N-1)N} + \frac{1}{N} \right] \\ &= 1 - \frac{1}{2s} + \frac{1}{2(N-1)} \frac{s}{N} - \frac{1}{2N(N-1)} . \end{aligned} \tag{4.3}$$

Take the value of  $s$  to be the optimum starting time  $s^*$  in (4.3), By allowing  $N$  to increase to infinity, we have:

$$\lim_{N \rightarrow \infty} P(X=1; s^*, N) = 1 . \tag{4.4}$$

This proves the theorem.

$P(X=1; s^*, N)$  approaches unity quite rapidly. Even for  $N$  as low as 12, the probability that exactly one object will be chosen exceeds 0.90.

Corollary 4.2:  $\lim_{N \rightarrow \infty} E_A(X; s^*, N) = 1.$

Proof: The corollary is a direct result of (4.4).

Corollary 4.3:  $\lim_{N \rightarrow \infty} [E_A(Y; s^*, N)/N] = 2/e .$

Proof: The result follows directly from (2.13) and Theorem 4.1.

## 5. SAMPLING UNTIL TWO (OR MORE) NON-CANDIDATES ARE ENCOUNTERED

One way to increase the probability of selecting the best object is to use the following procedure:

Procedure  $A_u$ : Select an integer  $u \leq N$ . Sample  $s-1$  objects without selecting any. Beginning with the  $s^{\text{th}}$  object, choose the first candidate that appears. Continue to sample objects, selecting candidates and rejecting non-candidates. The procedure is stopped if either the number of non-candidates sampled after the first choice was made is equal to  $u$  or the  $N^{\text{th}}$  candidate is sampled. If no choices have been made, the  $N^{\text{th}}$  object must be selected.

Denote the probability of success using  $A_u$  by  $P_{A_u}(CS; s, N)$  or by  $P_{A_u}(CS)$  when it is clear that  $s$  and  $N$  are given fixed values. The player is interested in choosing  $s$  to maximize  $P_{A_u}(CS)$ .

Using Procedure  $A_u$ , the player selects the best object in one of two mutually exclusive ways:

- (1) The first candidate is chosen at some time  $k \geq s$ .  
Up to  $u-2$  non-candidates appear between times  $k$  and  $j$ .  
At time  $j$ , the best object is chosen. All of the succeeding objects are non-candidates with probability one.



(2) Exactly  $u-1$  non-candidates appear between times  $k$  and  $j$ . The  $(j+1)^{\text{st}}$  object is a non-candidate with probability one.

The probability that event (1) of the preceding paragraph occurs is given by  $P_{A_{u-1}}(\text{CS})$ . If we then denote event (2) by  $W_{u-1}(s, k, j)$ , we have:

$$P_{A_u}(\text{CS}; s, N) = P_{A_{u-1}}(\text{CS}; s, N) + P(W_{u-1}(s)) . \quad (5.1)$$

To compute the last term in (5.1), first assume that  $u-1$  non-candidates appear at times  $m_1, m_2, \dots, m_{u-1}$ , and that  $s > 1$ . Then,

$$P(W_{u-1}(s, k, j, m_1, \dots, m_{u-1})) = \frac{s-1}{N} f(k-2, j-1) \prod_{i=1}^{u-1} (m_i - 1) , \quad (5.2)$$

where  $s \leq k \leq N-u$ ,  $k+u \leq j \leq N$ , and  $k < m_1 < m_2 < \dots < m_{u-1} < j$ .

Next, sum (5.2) on  $m_1, m_2, \dots, m_{u-1}$ :

$$P(W_{u-1}(s, k, j)) = \frac{s-1}{N} f(k-2, j-1) F(k, j) , \quad (5.3)$$

where

$$F(k, j) = \sum_{m_1=k+1}^{j-u+1} \sum_{m_2=m_1+1}^{j-u+2} \dots \sum_{m_{u-1}=m_{u-2}+1}^{j-1} \prod_{i=1}^{u-1} (m_i - 1) .$$

Summing (5.3) over  $k$  and  $j$  yields, for  $s > 1$ :

$$P(W_{u-1}(s)) = \frac{s-1}{N} \sum_{k=s}^{N-u} \sum_{j=k+u}^N f(k-2, j-1) F(k, j) . \quad (5.4)$$

When  $s=1$ , the calculations are similar:

$$P(W_{u-1}(1)) = \frac{1}{N} \sum_{j=u+1}^N f(1, j-1) F(1, j) . \quad (5.5)$$

Equation (5.1) can then be evaluated by forward iteration, beginning with equation (2.5) and then using equation (5.4) or (5.5).

While (5.1) does give an expression by which  $P_{A_u}(CS)$  can be calculated, it is not of much practical use. For  $u > 2$ , it is too time consuming to compute the equations by hand. It is possible to program (5.4) and (5.5), but this would also be difficult to do except for the smallest  $N$ , given the usual program size and time limitations at most installations.

When  $u=2$ , however, (5.1) can be put into a usable form, since  $W_1(s)$  can be written as a function of  $P_A(CS)$ . First, for  $s > 1$ , (5.3) takes the form:

$$P(W_1(s, k, j)) = \frac{s-1}{2N} [f(k-2, j-3) - f(k, j-1)] . \quad (5.6)$$

Then, by summing (5.6) over  $j$  and  $k$  we have:

$$P(W_1(s)) = \frac{1}{2} \frac{N-2}{N} P_A(CS; s, N-2) - \frac{1}{2} \frac{s-1}{s+1} P_A(CS; s+1, N) . \quad (5.7)$$

When  $s=1$ , (5.5) becomes:

$$P(W_1(1)) = \frac{1}{2} \frac{N-2}{N} P_A(CS; 1, N-2) . \quad (5.8)$$



Thus, for  $u=2$ , (5.1) can be written as:

$$P_{A_2}(CS; s, N) = P_A(CS; s, N) + \frac{1}{2} \frac{N-2}{N} P_A(CS; s, N-2) - \frac{1}{2} \frac{s-1}{s+1} P_A(CS; s+2, N) . \quad (5.9)$$

Table 2 contains the optimum starting time  $s^*$  and the probability of correct selection  $P_{A_2}(CS; s^*, N)$  for  $N=1(1)50(10)100, 1000$ . The values of  $s^*$  were found by calculating  $P_{A_2}(CS; s, N)$  for all  $s$  for a given  $N$ . As in section 3, the optimum value of  $s$  may be characterized.  $s^*$  is the smallest  $s$  such that:

$$2 > \sum_{k=s+2}^N \sum_{j=k}^N f(k-2, j-1) + \frac{1}{2} \sum_{k=s+2}^{N-2} \sum_{j=k}^{N-2} f(k-2, j-1) + \frac{1}{2} \frac{s-1}{s+1} - \sum_{j=s+3}^N f(s, j-1) - \frac{1}{2} \sum_{k=s+4}^N \sum_{j=k}^N f(k-2, j-1) . \quad (5.10)$$

Equation (5.10) is not of much practical use.

The optimum  $s$  for  $P_{A_2}(CS)$  heuristically appear to be equal to or at most one less than the optimum  $s$  for  $P_A(CS)$  for a given  $N$ . Note that for  $N=1000$ ,  $P_{A_2}(CS; 368, 1000) = .369462$  while  $P_A(CS; 368, 1000) = .368829$ . Thus, the player gains only a 0.17 percent advantage by using  $A_2$  instead of  $A$ . Under the assumption that  $s^*$  increases without bound, as seems to be the case, it can be shown in a manner similar to theorem 4.1 that  $A_2$

is asymptotically equivalent to the optimal procedure of the best choice Secretary Problem.

1. Optimal Starting Time, Probability of Success, Expected Number  
Selected and Expected Sample Size For Procedure A

N	s*	P <sub>A</sub> (CS)	E <sub>A</sub> (X)	E <sub>A</sub> (Y)	N	s*	P <sub>A</sub> (CS)	E <sub>A</sub> (X)	E <sub>A</sub> (Y)
1	1	1.0	1.0	1.0	29	11	.402284	1.042	21.67
2	1	1.0	1.5	2.0	30	11	.400795	1.042	22.02
3	1	.833333	1.667	2.500	31	12	.399903	1.038	23.40
4	2	.666667	1.250	3.667	32	12	.398877	1.038	23.76
5	2	.616667	1.283	4.083	33	13	.397655	1.035	25.12
6	2	.561111	1.297	4.367	34	13	.397000	1.035	25.50
7	3	.532143	1.171	5.725	35	13	.396056	1.035	25.86
8	3	.508929	1.179	6.071	36	14	.395189	1.032	27.23
9	4	.488029	1.121	7.392	37	14	.394555	1.032	27.60
10	4	.477447	1.126	7.774	38	14	.393682	1.033	27.96
11	4	.464790	1.129	8.113	39	15	.393078	1.030	29.33
12	5	.456275	1.096	9.475	40	15	.392468	1.030	29.70
13	5	.449412	1.099	9.842	41	15	.391657	1.030	30.06
14	5	.441306	1.101	10.18	42	16	.391252	1.028	31.43
15	6	.437435	1.080	11.56	43	16	.390667	1.028	31.80
16	6	.432542	1.081	11.92	44	17	.390051	1.026	33.16
17	7	.427892	1.066	13.27	45	17	.389657	1.026	33.54
18	7	.425041	1.068	13.65	46	17	.389095	1.027	33.90
19	7	.421316	1.069	14.01	47	18	.388646	1.025	35.27
20	8	.418630	1.058	15.37	48	18	.388252	1.025	35.64
21	8	.416291	1.059	15.74	49	18	.387713	1.025	36.00
22	8	.413320	1.060	16.09	50	19	.387395	1.024	37.37
23	9	.411764	1.051	17.47	60	22	.383985	1.020	44.04
24	9	.409792	1.052	17.84	70	26	.381708	1.017	51.72
25	10	.407709	1.046	19.19	80	30	.379950	1.015	59.40
26	10	.406477	1.046	19.57	90	34	.378554	1.013	67.07
27	10	.404778	1.047	19.93	100	37	.377497	1.012	73.75
28	11	.403405	1.041	21.30	1000	368	.368829	1.001	735.80



2. Optimal Starting Time and Probability of  
Success Using Procedure  $A_2$

N	s*	$P_{A_2}$ (CS)	N	s*	$P_{A_2}$ (CS)
1	1	1.0	29	11	.425286
2	1	1.0	30	11	.423490
3	1	1.0	31	11	.421380
4	1	.916667	32	12	.419751
5	1	.791667	33	12	.418168
6	2	.705556	34	12	.416328
7	2	.650000	35	13	.415159
8	3	.605060	36	13	.413746
9	3	.577745	37	14	.412348
10	3	.550033	38	14	.411287
11	4	.533805	39	14	.410015
12	4	.517892	40	15	.408956
13	5	.504109	41	15	.407980
14	5	.494550	42	15	.406825
15	5	.484139	43	16	.406027
16	6	.476812	44	16	.405124
17	6	.470002	45	17	.404159
18	7	.462872	46	17	.403471
19	7	.458410	47	17	.402631
20	7	.453265	48	18	.401874
21	8	.448759	49	18	.401221
22	8	.445204	50	18	.400438
23	8	.441144	60	22	.394941
24	9	.438201	70	26	.390975
25	9	.435280	80	29	.387992
26	10	.432080	90	33	.385756
27	10	.430016	100	37	.383939
28	10	.427558	1000	368	.369462

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